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Computational optimization for nonimaging solar concentrators using Generalized Pattern Search

B. Ilan*[a], C. Hoffman[a], C. Rimland[a]
[a]Applied Mathematics Department, School of Natural Sciences, University of California, Merced, CA 95343

ABSTRACT

We present a computational framework for optimizing nonimaging solar concentrators. Our approach is to represent the concentrator's shape as a polygon, use ray tracing to compute the flux at the receiver, and employ Generalized Pattern Search (GPS) on the polygon's vertices. Many shape optimization techniques use gradients to seek a direction of steepest ascent or descent. For solar concentrators, these approaches can easily get trapped in local minima. In contrast, GPS is a derivative-free method that seeks a global optimum on suitable meshes, without computing gradients. This helps to avoid getting trapped in local minima. Results for 2D concentrators show that our algorithm can converge to the ideal concentrator's shape as the number of polygon vertices increases. We also show that when the number of vertices is small and fixed, the optimal polygon can differ significantly from the polygon that would be obtained using a uniform collocation of the ideal shape. This approach could lead to a simple, accurate, and fast design method, and improve the performance and lower the fabrication costs of nonimaging concentrators for solar and thermal applications.

Keywords: nonimaging optics; shape optimization; solar concentrators

1. INTRODUCTION

1.1 Nonimaging solar concentrators

Nonimaging concentrators are designed to transfer as much light as possible from a source to a receiver[1]. The source and receiver can be points or extended objects, planar or non-planar. Nonimaging solar concentrators have been applied to solar-PV and solar-thermal applications. In addition, nonimaging optics is also used for illumination engineering[3]. For solar concentrators, the task of the optics is to allow for the widest acceptance angle resulting in the greatest collection of light.

There are few guiding principles for designing nonimaging concentrators, including the conservation of étendue and the edge-ray principle[1]. For two-dimensional (2D) concentrators, which can be used to design axially-symmetric 3D concentrators, there purely-analytical design methods, including the method of strings, the flow-line method[1,2]. However, for 3D concentrators, there is no theory or analytical technique for finding the optimal design.

Sophisticated computational approaches have been developed for these problems, such as the Simultaneous Multiple Surface (SMS) method[4] and the generalized functional method[5], which have been applied to design various 3D freeform nonimaging optics. Earlier, Ashdown[7] used ray-tracing coupled with genetic algorithms to design the reflector geometry of illuminating devices. Muschaweck et al. optimized over a family of parametric curves with a single or few degrees of freedom[8]. Daun et al.[9] applied gradient-based optimization techniques for designing radiant enclosures. Marston et al.[10] extended and modified this approach for designing 2D solar concentrators using Monte Carlo ray tracing and the Kiefer-Wolfowitz stochastic gradient-descent optimization method. Rukolaine[10] described a gradient-based technique to be used for determining optimal geometry for a 2D radiant enclosure problem. However, gradient-descent methods tend to get trapped in local minima, which can be far from optimal. Moreover, any local change in the shape of an optic will yield a large divergence of the light as it propagates. Therefore, gradients of the light flux at the receiver (for that light that reaches the receiver) give almost no meaningful information about the changes in the *local* shape of the concentrator. For this reason, we choose a different class of optimization techniques.

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2. ALGORITHMS

We seek to develop a computational framework that is relatively simple, accurate, fast, flexible and stable. To take a first step, we develop this approach for 2D concentrators. We represent the concentrator's shape (its top and bottom) as polygons and use deterministic ray-tracing. Each ray that reaches the concentrator is reflected from a perfect mirror and continues to propagate until it either reaches the receiver (where it is perfectly absorbed) or rejected, *i.e.*, exists from the aperture. The objective function is the relative number of rays that reaches the receiver, which is proportional to the flux at the receiver. To find the optimal positions of the polygons' vertices, we to use a derivative-free optimization technique known as pattern search.

2.1 Choosing the number of vertices and rays

Optimization techniques work better when there are fewer degrees of freedom. For this reason, we seed the algorithm (the initial shape of the concentrator) with a low-order polygon and increase the number of vertices as the shape converges. For simplicity, we describe the process for the top segment. We seed the algorithm with a 2-edge polygon, *i.e.,* a single vertex at the top, which is chosen at the center of the straight line that connects the top edges of the aperture and receiver. The pattern search algorithm (details below) seeks an optimal position for this vertex. When pattern search converges to a certain shape, the number of vertices is increased and a new seed shape for the next iterate is obtained by interpolation of the previous shape. The number of vertices is increased by two at each during the first few iterations and by 25% during advanced iterations. This conservative approach helps avoid getting stuck in local minima. For the interpolant, we use the Piecewise Cubic Hermite Interpolating Polynomial (PCHIP), which works well for preserving shapes[12].

For the ray tracing, the number of rays is increased with the number of polygonal edges, N_{edges}. Specifically, we choose uniformly distributed points along the source with $N_{sources} = \left\lceil \frac{3}{2} N_{edges} \right\rceil$, where $\lceil \cdot \rceil$ stands for the ceiling function. At each source point, we choose the number of angles as $N_{angles} = N_{sources}$, where the angles at each source point are uniformly distributed within the acceptance range (the directions that can enter the aperture). Therefore, the number of rays used in the ray-tracing for a trial shape scales as $N_{rays} = N_{sources} * N_{angles} \propto N_{edges}^2$. In this way, as the algorithm converges, the ray-tracing computation becomes more accurate. These choices yield a good trade-off between accuracy, speed, and stability.

2.2 Generalized Pattern Search

Pattern search is derivative-free optimization technique. A recent survey of such techniques can be found in [13]. Briefly, incumbent points (vertices in our problem) are polled from a prescribed mesh and a decision is made to update to an incumbent point when the objective function decreases (or increases). Then the mesh is contracted or expanded and the process repeats itself. The first technique of this type, called Coordinate Search, was developed by Fermi and Metropolis[14]. Hooke and Jeeves developed a more general approach[15], which they coined "pattern search", also known as Generalized Pattern Search (GPS). Torczon and collaborators[16-21] provided the first theoretical basis for pattern search, introduced the generating set search (GSS) class of algorithms, dynamic adaptation, and parallelization. Different algorithms differ on how they prescribe the mesh. GPS uses a positive basis of unit vectors. GSS resembles GPS, but adapts the mesh to account for linear constraints. These techniques are available in MATLAB, which we use. One polling method that works consistently well for our problems is GSS Positive Basis 2N.

2.3 Convexity constraint

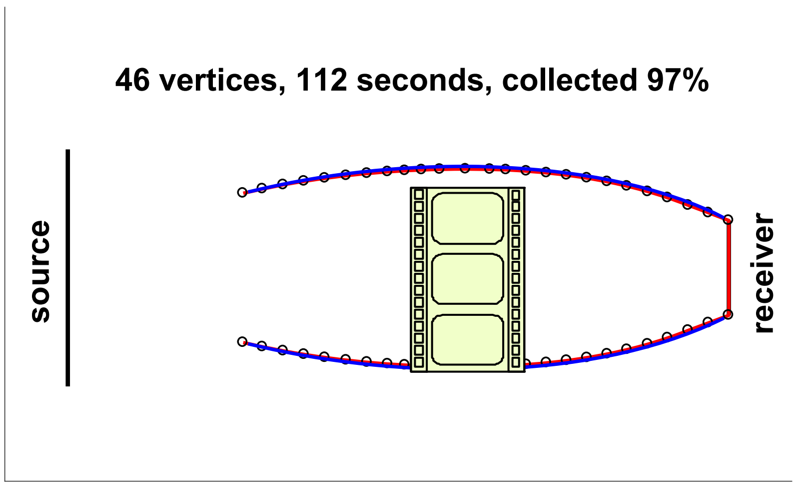
Because the optimal shape is strictly convex, we try to enforce strict convexity. Specifically, let $(x_{i,}, y_i)$ be the position of the *i*'th vertex, where *i=0* and $i = N_{segments}$ correspond to the top edges of the aperture and receiver, respectively. Let us denote the difference between successive vertices as $(\Delta x_{i,}, \Delta y_i) = (x_{i,}, y_i) - (x_{i-1,}, y_{i-1})$. The shape of the concentrator is strictly convex if, and only if, all the determinants $d_i = \Delta x_{i-1} \Delta y_i - \Delta x_i \Delta y_{i-1}$ are negative. To this end, we define the constant $c = \frac{1}{2} - all(d_i < 0)$, which returns $-\frac{1}{2}$ if the shape is convex and $+\frac{1}{2}$ otherwise. Convexity is a nonlinear constraint, which can be hard to enforce. Although convexity could be cast as an equality constraint, we find that it works better to cast it as an inequality constraint, *i.e.,* $c < 0$, using the Augmented Lagrangian Pattern Search method[19]. This "soft" enforcement approach means that the resulting shape has an incentive to be convex, but can deviate from this constraint.

3. RESULTS

3.1 Planar receiver

The first problem we consider is when the source, receiver, and aperture are planar (line segments in 2D), which are parallel to each other. The aperture's position is chosen using the principle of conservation of étendue. The ideal concentrator is known and can be obtained using the string method. Specifically, the top portion of the ideal concentrator is a segment of an ellipse, whose foci are the bottom edges of the source and the receiver, and which passes through the top edges of the aperture and receiver. The bottom portion is obtained symmetrically.

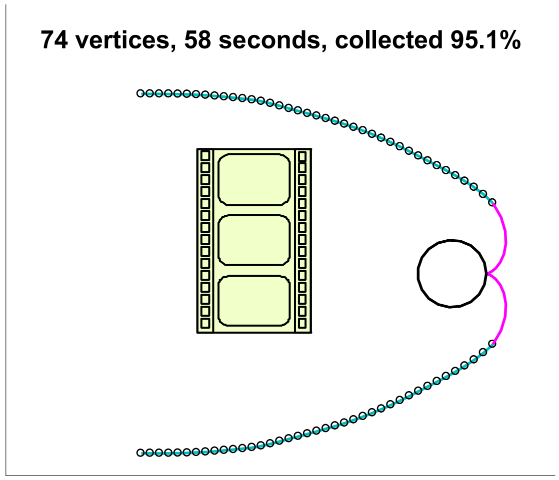
Video 1 shows the convergence of the algorithm to the ideal shape. The algorithm starts with straight lines connecting between the aperture and receiver. The process is stopped when the concentrator collects 97% of the rays, when the resulting polygon has 46 vertices and the ray-tracing uses 841 rays. For all practical purposes, the optimized shape is the same as the ideal one. This process takes 112 wall-clock seconds on a laptop computer. We have verified that this convergence remains stable when the various user-defined parameters (polling method, mesh expansion / contraction parameters, number of rays, *etc*.) are varied.



Video 1. [planar_absorber_movie.mp4]. A concentrator for a planar receiver (line segment on the right side). The optimization algorithm converges to the ideal shape (blue curves), reaching 97% collection efficiency using 46 vertices. The wall-clock runtime (seconds) is shown. http://dx.doi.org/10.1117/12.2503895.1

3.2 Cylindrical receiver

Non-planar absorbers can have various advantages over planar absorbers for solar concentration[1]. Here we consider a cylindrical trough design, where the 2D cross section of the receiver is a circle. The optimal design, which can be found using the string method[1,21], consists of an involute (see the figure in Video 2). The section of the involute to the right of the receiver is an involute of the receiver's circle, which has a cusp point at the right-edge of this circle. That section is kept fixed during the optimization process. The optimization algorithm is applied to the top part of the concentrator. Future work will develop a method to optimize the entire shape. As in the previous example, the algorithm starts with straight lines connecting between the aperture and the beginning of the circle's involute. The algorithm converges to an almost ideal shape, reaching 95% collection efficiency using 74 vertices.



Video 2. [cylindrical_absorber_movie.mp4]. A concentrator for a cylindrical receiver (black circle). The section (in magenta) to the right of the receiver is kept fixed. The optimization algorithm is applied to the top segment and reaches 95% collection efficiency using 74 vertices. http://dx.doi.org/10.1117/12.2503895.2

3.3 Low-degree polygonal concentrator

From a manufacturing perspective, it is easier and cheaper to make a concentrator whose shape is a low-degree polygon than any freeform curve. This raises the question: what is the optimal low-degree polygonal concentrator? A plausible, intuitive answer is to consider the ideal shape and collocate uniformly-spaced vertices on it. We call this the *ideal-collocation design*. Theoretically, as the number of vertices is increased, the resulting concentrator would approach the ideal one. However, there is no reason why the ideal-collocation design is optimal among the class of all low-degree polygons.

To investigate this, we consider the simplest case: a single vertex polygon. We seed the computational algorithm (using 100 source points for accuracy) with the ideal-collocation design, i.e., a vertex point at the center of the ideal (ellipse) curve. The algorithm converges to an optimized design. Figure 1 shows this optimized design, which yields 88% collection, compared with 77% using the ideal-collocation design. It is remarkable that the optimized design is quite different from the ideal-collocation design: its vertex neither lies on the ideal curve (part of an ellipse) nor is it equally distant between the aperture and receiver. We have tested that this optimized shape is stable by using different seeds and algorithmic parameters. For this reason, we believe that it is optimal. This shows that the optimized design is significantly better than the ideal-collocation design.

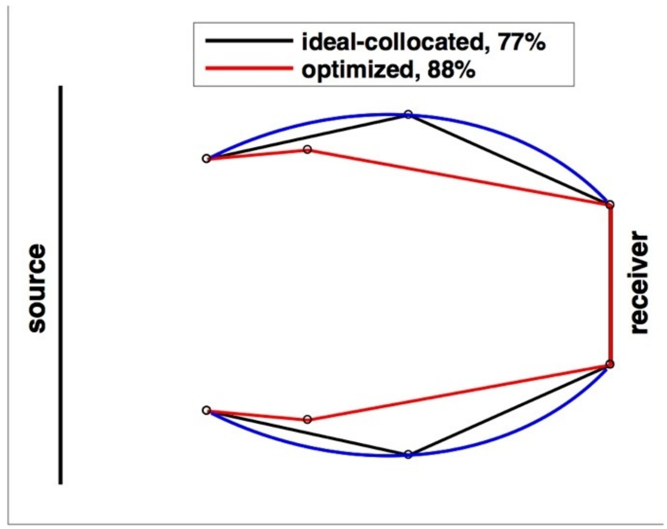


Figure 1. The optimized concentrator design using a single-vertex polygon (red) yields 88% collection. In comparison, the ideal-collocation design (black), which is obtained by placing the vertex at the center point of the ideal shape (blue), yields 77% collection.

4. CONCLUSIONS

Computational optimization using pattern search is a viable approach for designing optimal nonimaging solar concentrators.

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